Conclusions

In this Note, we consider the stabilization of large, flexible structures with mode residualization. A new frequency domain stability condition for a high-order structure controlled by a reduced-order observer-based controller is derived. As linear parametric perturbations are involved in the controlled structure, an additional stability condition that guarantees robust stability for the perturbed structure is also presented.

Appendix: Proof of Theorem 1

Taking Laplace transform of the state-space representations of Eq. (5) yields

$$sZ_1(s)=H_{11}Z_1(s)+z_1(0)+H_{12}X_2(s) \tag{A1a}$$

$$X_2(s)=(sI-H_{22})^{-1}H_{21}Z_1(s)+(sI-H_{22})^{-1}x_2(0) \tag{A1b}$$

$$Y(s)=\tilde{C}_1Z_1(s)+C_2X_2(s) \tag{A1c}$$

where $z_1(0)$ and $x_2(0)$ are, respectively, the initial states of $z_1(t)$ and $x_2(t)$. Substituting Eq. (A1b) into Eq. (A1a), and after some manipulations, we get

$$Z_1(s)=K(s)^{-1}z_1(0)+K(s)^{-1}H_{12}(sI-H_{22})^{-1}x_2(0) \tag{A2}$$

where $K(s)=sI-H_{11}-H_{12}(sI-H_{22})^{-1}H_{21}$. $K(s)$ can be further expressed as

$$\begin{aligned}K(s)&=sI-H_{11}+H_{12}H_{22}^{-1}H_{21}-H_{12}H_{22}^{-1}H_{21}\\&\quad-H_{12}(sI-H_{22})^{-1}H_{21}\\&=sI-\Lambda-H_{12}H_{22}^{-1}[I-(I-sH_{22}^{-1})^{-1}]H_{21}\\&\qquad(\text{where }\Lambda=H_{11}-H_{12}H_{22}^{-1}H_{21})\\&=(sI-\Lambda)[I-s(sI-\Lambda)^{-1}H_{12}H_{22}^{-1}(sI-H_{22})^{-1}H_{21}]\end{aligned} \tag{A3}$$

Substituting Eq. (A3) into Eq. (A2), we obtain

$$\begin{aligned}Z_1(s)&=[I-s\,(sI-\Lambda)^{-1}H_{12}H_{22}^{-1}(sI-H_{22})^{-1}H_{21}]^{-1}\\&\quad\times(sI-\Lambda)^{-1}z_1(0)+[I-s(sI-\Lambda)^{-1}H_{12}H_{22}^{-1}\\&\quad\times(sI-H_{22})^{-1}H_{21}]^{-1}(sI-\Lambda)^{-1}H_{12}(sI-H_{22})^{-1}x_2(0)\end{aligned} \tag{A4}$$

Since

$$(sI-\Lambda)^{-1}\in S^{2n1\times 2n1},\qquad (sI-H_{22})^{-1}\in S^{n2\times n2}$$

$$s(sI-\Lambda)^{-1}H_{12}H_{22}^{-1}(sI-H_{22})^{-1}H_{21}\in S^{2n1\times 2n1}$$

by Lemmas 1 and 2, we therefore know that if

$$\bar{\sigma}[j\omega(j\omega I-\Lambda)^{-1}H_{12}H_{22}^{-1}(j\omega I-H_{22})^{-1}H_{21}]<1,\quad \text{for all } \omega\geq 0$$

holds, then $z_1(t)$, the inverse Laplace transform of $Z_1(s)$ in Eq. (A4), is asymptotically stable. From Eq. (A1b), since $(sI-H_{22})^{-1}\in S^{n2\times n2}$ and $z_1(t)$ and $x_2(t)$ have been stabilized. Since both $z_1(t)$ and $x_2(t)$ are stabilized, obviously, $y(t)$ is also asymptotically stable.

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Effect of Thrust/Speed Dependence on Long-Period Dynamics in Supersonic Flight

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Nomenclature

$A(s)$	= coefficient matrix of the homogenous system
B	= scaling matrix for control inputs
C_D	= drag coefficient
C_{D_u}	= $\partial C_D/\partial(u/V_0)$
C_{D_α}	= $\partial C_D/\partial\alpha$
C_L	= lift coefficient
C_{L_u}	= $\partial C_L/\partial(u/V_0)$
C_{L_α}	= $\partial C_L/\partial\alpha$
C_m	= pitching moment coefficient
C_{m_q}	= pitch damping, $=2\partial C_m/\partial(q\bar{c}/V_0)$
C_{m_α}	= $\partial C_m/\partial\alpha$
$C_{m_{\dot\alpha}}$	= $2\partial C_m/\partial(\dot\alpha\bar{c}/V)$
$\bar{c}$	= mean aerodynamic chord
g	= acceleration due to gravity
h	= altitude
i_y	= radius of gyration
k_ρ	= $-(g/V_0^2)/\rho_h$
M	= Mach number
n_u	= thrust/speed dependence, $=(V_0/T_0)\partial T/\partial u$
$\bar{q}$	= dynamic pressure, $=(\rho/2)V^2$
S	= reference area
s	= Laplace operator
T	= thrust
u	= speed perturbation
V	= airspeed
x	= variable vector
α	= angle of attack
δ_T	= thrust setting
μ	= relative mass parameter, $=2m/(\rho S\bar{c})$
ρ	= air density
ρ_h	= density gradient, $=(1/\rho_0)\,d\rho/dh$
σ	= real part of complex variable

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τ = reference time, $=\bar{c}/V_0$
ω = imaginary part of complex variable
ω_n = natural frequency

Introduction

It is well known that thrust changes caused by speed perturbations exert a significant influence on the phugoid in subsonic flight.[1,2] This effect may be stable or unstable, depending on the thrust slope with speed. A positive thrust slope yields a destabilization whereas the opposite is true for a negative thrust slope. According to the effect of inherent thrust changes with speed, a well-known means for improving the dynamic stability of aircraft is the autothrottle, which is a basic control concept using speed feedback to the throttle.[1,2] This particularly concerns the long-period dynamics and provides the ability for an efficient stabilization of the phugoid mode of motion. In Fig. 1, it is shown in which way the phugoid eigenvalues are changed and how efficient the auto throttle is as a stabilization device for subsonic flight.

In supersonic flight, the long-term stability may show only marginal characteristics (e.g., see Ref. 3). This particularly concerns the phugoid. Thus, a particular need can exist for a stability augmentation system for which an autothrottle may be considered as an efficient candidate due to the good experience gained with this device. Furthermore, advanced air-breathing propulsion systems for supersonic aircraft show significant variations in thrust with Mach number.[4] These inherent thrust variations are generally considered to have a significant influence on the phugoid.

It is the purpose of this Note to show that thrust variations with speed (or Mach number) have practically no effect on the phugoid in supersonic flight. This particularly concerns a positive thrust slope ($\partial T/\partial u > 0$ or $\partial T/\partial M > 0$), which is usually considered as strongly destabilizing the phugoid. It will be shown that this effect is not existent. Rather, the phugoid is not influenced at all in supersonic flight. Similarly, it will be shown that an autothrottle provides no means for improving phugoid stability.

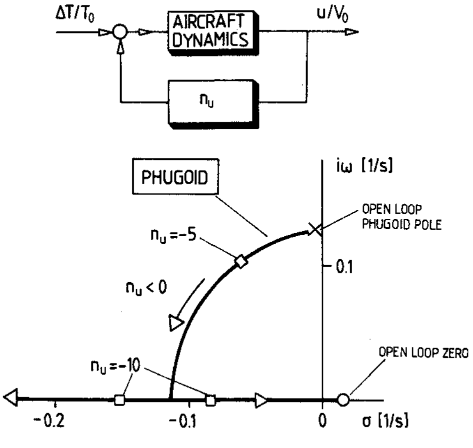

Fig. 1 Effect of speed feedback to thrust on long-term dynamics in subsonic flight ($M = 0.25$).

The results of this Note may also be of interest as regards the experience gained with an autothrottle control system applied to a Mach 3 aircraft.[5] The fact that this system has been successfully operated is not in contradiction with the results of the present Note because it has been applied in combination with other feedback loops. Rather, the effects that an autothrottle control provides when operating alone may help one to understand why it can be applied successfully in combination with other feedback loops. Furthermore, the autothrottle control is an important basic feedback concept the characteristics of which should be understood. This particularly holds for the case considered, which shows a significant reduction of autothrottle effectiveness for supersonic flight when compared with the well-known and efficient autothrottle characteristics in subsonic flight.

Aircraft Dynamics

In supersonic flight, aerodynamic forces and moments due to altitude perturbations (density changes) exert an influence on dynamic stability.[4,5] Accordingly, they have to be accounted for in modeling aircraft dynamics. The resulting linearized equations of motion for perturbations from a horizontal reference flight condition may be expressed as

$$A(s)x(s) = -B\delta_T(s) \qquad \text{(1a)}$$

where

$$x(s) = [u/V_0 \quad \Delta\alpha \quad \Delta h/\bar{c}]^T \qquad \text{(1b)}$$

$$A(s) = \begin{bmatrix} s\mu\tau + [2 + (C_{D_u}/C_D) - n_u]C_D & C_{D_\alpha} & s\tau C_L \\ 2C_L + C_{L_u} & C_{L_\alpha} + C_D & -\mu(s\tau)^2 + \bar{c}\rho_h C_L \\ 0 & -\mu(s\tau)^2(i_y/\bar{c})^2 & -\mu(s\tau)^3(i_y/\bar{c})^2 \\ & +s\tau(C_{m_q} + C_{m_{\dot\alpha}})/2 + C_{m_\alpha} & +(s\tau)^2 C_{m_q}/2 \end{bmatrix} \qquad \text{(1c)}$$

$$B(s) = [T_0/(\bar{q}_0 S) \quad 0 \quad 0]^T \qquad \text{(1d)}$$

The effect of thrust/speed dependence is treated by applying the root locus technique. Therefore, thrust changes due to speed perturbations may be understood as produced by a hypothetical control loop using speed feedback to thrust (as indicated by the block diagram in the upper part of Fig. 3). This reasoning may be applied to engine characteristics showing inherent thrust changes with speed or to an autothrottle system artificially introducing thrust changes.

The root locus of such a control loop is determined by its open-loop poles and zeros. The open-loop poles represent the eigenvalues of an aircraft, showing no thrust changes with speed. In supersonic flight, there are five open-loop poles, which are associated with three modes of motion. They may be identified as 1) short-period mode (two complex poles: σ_α, ω_{n_α}), 2) phugoid (two complex poles of small magnitude: σ_p, ω_{n_p}), and 3) height mode (real pole of small magnitude: s_H).

It is not necessary for the following considerations to present any details or approximate expressions for the poles. Rather, their relative location to the open-loop zeros is of interest. There are two complex pairs of zeros (σ_1, w_{n_1} and σ_2, w_{n_2}) in supersonic flight (see the Appendix). Their relative location to the open-loop poles may be expressed as

$$\sigma_1 \approx \sigma_\alpha, \qquad \omega_{n_1} \approx \omega_{n_\alpha} \qquad \text{(2)}$$

$$\sigma_2 \approx (\sigma_p)_{\partial T/\partial u = 0}, \qquad \omega_{n_2} \approx \sqrt{1 - k_p}\, \omega_{n_p} \qquad \text{(3)}$$

According to Eq. (2), the poles (σ_α, ω_{n_α}) and zeros (σ_1, ω_{n_1}) effectively cancel each other so that the short-period characteristics remain practically unchanged. This result, which is

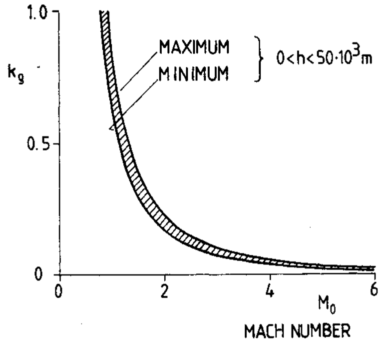

Fig. 2 The $k\rho$ as a function of Mach number.

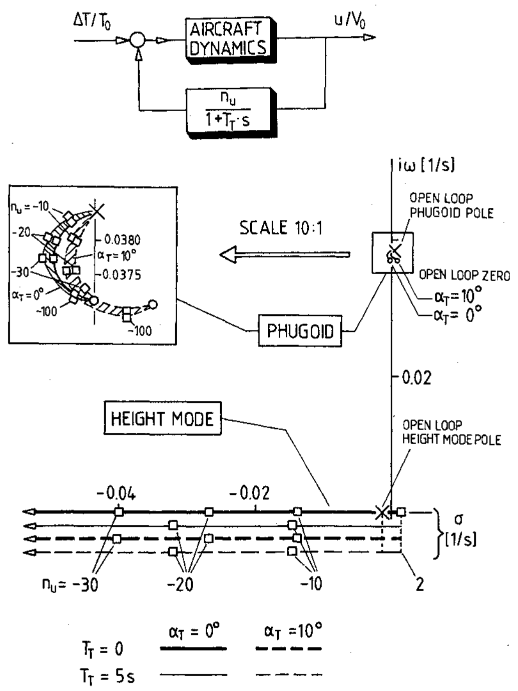

Fig. 3 Effect of speed dependent thrust changes on long-period dynamics in supersonic flight ($M=3$).

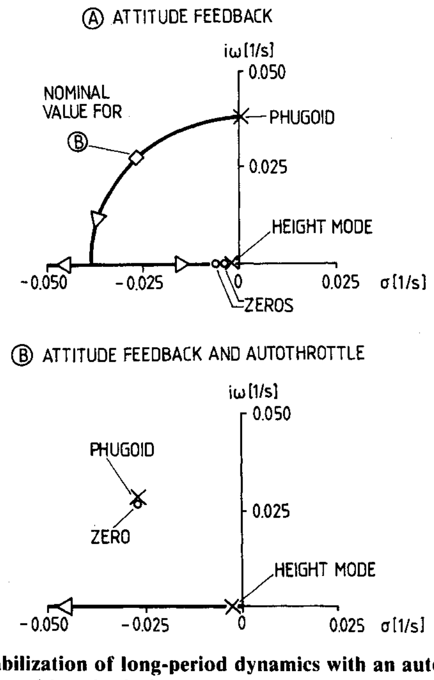

Fig. 4 Stabilization of long-period dynamics with an autothrottle in combination with attitude feedback.

well known, agrees with aircraft characteristics in subsonic and transonic flight. However, the correlation described by Eq. (3) for the phugoid and the remaining zeros is new and differs completely from the relationship in subsonic flight. According to Eq. (3), a close relation between these poles and zeros exists. This is because

$$k_\rho = -\frac{\rho_0}{\mathrm{d}\rho/\mathrm{d}h}\frac{g}{V_0^2} \ll 1 \tag{4}$$

holds throughout the whole supersonic Mach number region (see Fig. 2). This close relation is decisive for the root locus because it yields an effective cancellation. Thus, the phugoid remains practically unchanged.

Results

For a Mach 3 supersonic transport, the effect of thrust/speed dependence on long-term dynamics is illustrated in Fig. 3. The main result concerns the root locus emerging from the open-loop phugoid poles. It is completely bounded by the close location of the zeros resulting in an effective cancellation as just described. As a result, the phugoid is practically not influenced. This holds for negative n_u values used in Fig. 3 for demonstrating a possible autothrottle application as well as for positive values, which may be considered an inherent engine characteristic existing in supersonic flight ($n_u = 2$ is used in Fig. 3). Comparison of Fig. 3 with Fig. 1 makes evident the contrast to the relationship existing in subsonic flight.

The only effect of thrust/speed dependence concerns the height mode, which is significantly influenced.

The feedback gain n_u applied in Fig. 3 may be used for describing inherent thrust changes with speed according to the following expression for thrust slope

$$\frac{\partial T}{\partial u} = \frac{T_0}{V_0} n_u \tag{5}$$

Values of turbojet-type engines showing inherent thrust variations with speed/Mach number in supersonic flight may be as large as $n_u = 2$.

Such a value implying a positive slope is generally considered as significantly destabilizing the phugoid. From Fig. 3, it follows that the phugoid shows practically no changes. However, the height mode is destabilized. Thus, an aperiodic divergence is introduced by a positive thrust slope in supersonic flight.

It may be of interest to note that a thrust vector inclination or a time delay in engine response characteristics have an insignificant effect only. This is also illustrated in Fig. 3. Concerning a possible autothrottle application, it follows from Fig. 3 that such a device is ineffective in regard to overall long-period stability since phugoid stability characteristics are not improved.

Concluding Remarks

The results presented show that the autothrottle as a basic feedback loop cannot improve the overall stability in supersonic flight. Its only effect is restricted to the height mode. However, this effect of the autothrottle may be used to improve the overall stability when it is combined with another feedback. In such a way, the autothrottle was successfully applied to a Mach 3 aircraft.[5] An example for an autothrottle application in combination with another feedback loop is shown in Fig. 4. Here, attitude feedback to the pitch control surface is used for phugoid stabilization (Fig. 4a). Since the height mode shows practically no changes due to attitude feedback, an autothrottle control may be additionally applied for height-mode stabilization (Fig. 4b). It is of interest to note that the autothrottle has practically no effect on the phugoid in this case, too.

Appendix

By expanding the A matrix with the B matrix as its first column, the following approximate expressions for the zeros may be derived (neglecting unimportant contributions)

$$\omega_{n_1}^2 \approx [V_0/\mu\bar{c})]^2(\bar{c}/i_y)^2[\mu C_{m_\alpha} + C_{L_\alpha}(C_{m_q} + C_{m_{\dot\alpha}})/2]$$

$$\sigma_1 \approx V_0/(2\mu\bar{c})[C_{L_\alpha} - (\bar{c}/i_y)^2(C_{m_q} + C_{m_{\dot\alpha}})/2] \tag{A1}$$

$$\omega_{n2}^2 \approx -C_L V_0^2 \rho_h/(\mu\bar{c}) = -g\rho_h$$

$$\sigma_2 \approx 0 \tag{A2}$$

Equations (A1) agree with the well-known approximations for the short-period mode (σ_α, ω_{n_α}), thus resulting in Eqs. (2). With regard to Eqs. (A2), the following phugoid approximations resulting from expanding the A matrix may be considered

$$\omega_{n_p}^2 \approx -g\rho_h + \left(2+\frac{C_{L_u}}{2C_L}\right)\left(\frac{g}{V_0}\right)^2 = -g\rho_h\left[1-\left(2+\frac{C_{L_u}}{2C_L}\right)k_\rho\right]$$

$$\sigma_p \approx \left(1+\frac{C_{L_u}}{2C_L}\right)k_\rho n_u \frac{g}{V_0}\frac{C_D}{C_L} \tag{A3}$$

Combining these expressions with Eqs. (A2), the correlation described by Eqs. (2) results.

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Generalized Gradient Algorithm for Trajectory Optimization

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Introduction

EQUALITY constraints represent a general class of path constraints in trajectory optimization,[1] since many types of constraints can be converted into path equality constraints. Miele[1] has developed an algorithm named Sequential Gradient Restoration Algorithm (SGRA),[1] which can solve general trajectory optimization problems. Goh and Teo[2] proposed a unified control parameterization approach that converts an optimal control problem into a parameter optimization problem. These algorithms employ a sequence of cycles, each cycle having two phases. One phase improves the performance index while the other reduces the constraint violations.

In this Note, a generalized gradient is found that improves the performance index and reduces the constraints at the same time.

Problem Statement

The general problem that will be considered has the following form:

$$\min_{u,\pi} I = \phi[x(1),\pi] + \int_0^1 L(x, u, \pi, \tau)\, d\tau \tag{1}$$

subject to

$$\dot{x} = f(x, u, \pi, \tau) \tag{2}$$

and $x(0)$ is given with the following constraints:

$$\psi[x(1),\pi] = 0 \tag{3}$$

$$S(x, u, \pi, \tau) = 0 \tag{4}$$

where u is the $m \times 1$ control vector, x is the $n \times 1$ state vector, π is the $p \times 1$ parameter vector, τ is the normalized time in (0,1), ψ is the $q \times 1$ terminal constraint vector, and S is the $r \times 1$ path equality constraint vector. A meaningful problem requires: $r \le m$ and $q \le n + p^* \le n + p$, where p^* is the number of parameters in ψ.[1]

Let us first consider the problem without terminal constraints. If there is no path equality constraint, the problem can be handled in the conventional way:

$$J = \phi + \int_0^1 \left[L + \lambda^T(f - \dot{x})\right] d\tau$$

$$= (\phi - \lambda^T x)_1 + (\lambda^T x)_0 + \int_0^1 (L + \lambda^T f + \dot{\lambda}^T x)\, d\tau$$

Define

$$H \triangleq L + \lambda^T f$$

The first variation of the augmented cost functional is then

$$\delta J = (\phi_x - \lambda^T)_1 \delta x_1 + \left[(\phi_\pi)_1 + \int_0^1 H_\pi\, d\tau\right]\delta\pi + \int_0^1 \left[(H_x + \dot{\lambda}^T)\delta x + H_u \delta u\right] d\tau \tag{5}$$

When path constraints exist, the following r equations must be satisfied to maintain the path constraints to first order:

$$S_x \delta x + S_u \delta u + S_\pi \delta\pi = 0 \tag{6}$$

If the path equality constraints contain control components, $|S_u S_u^T| \neq 0$ for $0 \le \tau \le 1$. The general solution of the algebraic equation, Eq. (6), is

$$\delta u = -D_u S_x \delta x - D_u S_\pi \delta\pi + \delta^* u \tag{7}$$

where $\delta^* u$ is the homogeneous part: $S_u \delta^* u = 0$ and $D_u \triangleq S_u^T (S_u S_u^T)^{-1}$.

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